

SUMS/PRODUCTS OF TRIG FUNCTIONS, AND PROOFS

$$\begin{aligned}2 \sin A \cos B &= \sin(A + B) + \sin(A - B) & \sin A + \sin B &= 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\2 \cos A \sin B &= \sin(A + B) - \sin(A - B) & \sin A - \sin B &= 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \\2 \cos A \cos B &= \cos(A + B) + \cos(A - B) & \cos A + \cos B &= 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\2 \sin A \sin B &= \cos(A - B) - \cos(A + B) & \cos A - \cos B &= -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\end{aligned}$$

Other useful identities:

$$\begin{aligned}\bullet \sin(-x) &= -\sin x & \bullet \sin x &= \cos\left(\frac{\pi}{2} - x\right) = \cos(90^\circ - x) \\ \bullet \cos(-x) &= \cos x & \bullet \cos x &= \sin\left(\frac{\pi}{2} - x\right) = \sin(90^\circ - x)\end{aligned}$$

Examples:

1) Write $2 \sin 45^\circ \cos 15^\circ$ as a sum.

$$2 \sin 45^\circ \cos 15^\circ = \sin(45^\circ + 15^\circ) + \sin(45^\circ - 15^\circ) = \sin 60^\circ + \sin 30^\circ$$

2) Show that $\cos 100^\circ + \cos 20^\circ = \cos 40^\circ$.

$$\begin{aligned}\cos 100^\circ + \cos 20^\circ &= 2 \cos\left(\frac{100^\circ + 20^\circ}{2}\right) \cos\left(\frac{100^\circ - 20^\circ}{2}\right) \\ &= 2 \cos 60^\circ \cos 40^\circ = 2 \cdot \frac{1}{2} \cdot \cos 40^\circ = \cos 40^\circ\end{aligned}$$

3) Prove that $\frac{\sin 8A + \sin 4A}{\cos 8A - \cos 4A} = -\cot 2A$

$$\begin{aligned}\text{LHS} &= \frac{\sin 8A + \sin 4A}{\cos 8A - \cos 4A} = \frac{2 \sin\left(\frac{8A+4A}{2}\right) \cos\left(\frac{8A-4A}{2}\right)}{-2 \sin\left(\frac{8A+4A}{2}\right) \sin\left(\frac{8A-4A}{2}\right)} = \frac{2 \sin 6A \cos 2A}{-2 \sin 6A \sin 2A} \\ &= \frac{-\cos 2A}{\sin 2A} \\ &= -\cot 2A \\ &= \text{RHS}\end{aligned}$$

Delta Ex 34.9 pg 328 Q 6, 10, 13, 16

Delta Ex 34.10 pg 330 Q 8, 9, 12, 15 onwards